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## Improving Bounds on Penguin Pollution in $B \rightarrow \pi\pi$

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### Abstract

In the presence of penguin contributions, the indirect CP asymmetry in  $B^0(t) \rightarrow \pi^+\pi^-$  measures  $\sin(2\alpha + 2\theta)$ , where  $2\theta$  parametrizes the size of the penguin “pollution.” We derive a new upper bound on  $|2\theta|$ , requiring the measurement of  $BR(B^+ \rightarrow \pi^+\pi^0)$  and an upper bound on  $B^{00} \equiv \frac{1}{2}[BR(B^0 \rightarrow \pi^0\pi^0) + BR(\overline{B}^0 \rightarrow \pi^0\pi^0)]$ . The new bound is stronger than those previously discussed in the literature. We also present a lower bound on  $B^{00}$ . Current data may suggest that it is not very small, in which case  $\theta$  can be determined using a complete isospin analysis.

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Over the past decade or so, a great deal of attention has been focussed on CP violation in the  $B$  system. By measuring  $\alpha$ ,  $\beta$  and  $\gamma$ , the three interior angles of the unitarity triangle, it will be possible to test the standard model (SM) explanation of CP violation [1]. Indeed, the first measurements of  $\beta$  have already been reported [2], and it is hoped that we will soon have definitive evidence of CP violation in  $B$  decays.

For the measurement of the angle  $\alpha$ , a principal decay mode considered is  $B^0(t) \rightarrow \pi^+\pi^-$ . (The decays  $B^0(t) \rightarrow \rho\pi \rightarrow \pi^+\pi^-\pi^0$  [3] and  $B_{d,s}^0(t) \rightarrow K^{(*)}\bar{K}^{(*)}$  [4] can also be used to cleanly obtain  $\alpha$ .) Unfortunately, this mode suffers from a well-known problem: penguin contributions may be large [5], and their presence will spoil the clean extraction of  $\alpha$ . This problem of penguin “pollution” can be eliminated with the help of an isospin analysis [6]. By measuring the rates for  $B^+ \rightarrow \pi^+\pi^0$  and  $B^0/\bar{B}^0 \rightarrow \pi^0\pi^0$ , in addition to  $B^0(t) \rightarrow \pi^+\pi^-$ , the penguin contributions can be eliminated so that  $\alpha$  can again be measured cleanly.

However, the isospin analysis itself suffers from a potential practical complication: it requires separate measurements of  $BR(B^0 \rightarrow \pi^0\pi^0)$  and  $BR(\bar{B}^0 \rightarrow \pi^0\pi^0)$ . This may be a problem for several reasons. First, these branching ratios are expected to be smaller than  $B^0 \rightarrow \pi^+\pi^-$ . Second, the presence of two  $\pi^0$ 's in the final state means that the reconstruction efficiency is also smaller. And third, in order to measure the two branching ratios individually, it will be necessary to tag the decaying  $B^0$  or  $\bar{B}^0$  meson, which will further reduce the measurement efficiency. The upshot is that it may not be possible to measure either of these two branching ratios, or we may only have information (i.e. an actual measurement or an upper limit) on the sum of the branching ratios. In either case, a full isospin analysis cannot be carried out.

But this then begs the question: assuming that we have, at best, only partial knowledge of the sum of  $BR(B^0 \rightarrow \pi^0\pi^0)$  and  $BR(\bar{B}^0 \rightarrow \pi^0\pi^0)$ , can we at least put bounds on the size of penguin pollution? To be more precise: in the presence of penguin amplitudes, the CP asymmetry in  $B^0(t) \rightarrow \pi^+\pi^-$  does not measure  $\sin 2\alpha$ , but rather  $\sin(2\alpha + 2\theta)$ , where  $2\theta$  parametrizes the effect of the penguin contributions. Is it possible to constrain  $\theta$ ? As demonstrated by Grossman and Quinn [7], the answer to this question is yes. They were able to show that  $|2\theta|$  can be bounded even if we have only an upper limit on the sum of  $BR(B^0 \rightarrow \pi^0\pi^0)$  and  $BR(\bar{B}^0 \rightarrow \pi^0\pi^0)$ . Charles [8] also examined this question, and found an improvement to the Grossman-Quinn bound involving the direct asymmetry in  $B^0 \rightarrow \pi^+\pi^-$ , as well as an independent bound involving different measurements.

The main purpose of this Letter is to present a new bound on  $|2\theta|$  which is an improvement on both the Grossman-Quinn and Charles bounds. In contrast to the earlier bounds, the new bound follows from the requirements that the two isospin triangles close and have a common base, making it the most stringent bound possible on  $|2\theta|$ . Indeed, the new bound contains the two previous bounds as limiting cases. We also present the constraints on the sum of  $BR(B^0 \rightarrow \pi^0\pi^0)$  and  $BR(\bar{B}^0 \rightarrow \pi^0\pi^0)$  which follow from the requirement of closure of the triangles. As we will show, if  $BR(B^+ \rightarrow \pi^+\pi^0)/BR(B^0 \rightarrow \pi^+\pi^-)$  is larger than one, as present experimental central values suggest, the branching ratios for  $B^0/\bar{B}^0 \rightarrow \pi^0\pi^0$  cannot be tiny. In this case, it may well be possible to carry out the full isospin analysis. Finally, we

show how measurements of  $B^0(t) \rightarrow \pi^+\pi^-$  alone can be used to place a lower limit on the magnitude of the penguin amplitude.

We begin with a brief review of the bounds of Grossman-Quinn and Charles. Defining

$$\begin{aligned} B^{+-} &\equiv \frac{1}{2} (|A^{+-}|^2 + |\bar{A}^{+-}|^2) , \\ a_{dir}^{+-} &\equiv \frac{|A^{+-}|^2 - |\bar{A}^{+-}|^2}{|A^{+-}|^2 + |\bar{A}^{+-}|^2} , \\ B^{00} &\equiv \frac{1}{2} (|A^{00}|^2 + |\bar{A}^{00}|^2) , \\ B^{+0} &\equiv |A^{+0}|^2 , \end{aligned} \tag{1}$$

where  $A^{+-}$  and  $\bar{A}^{+-}$  are the amplitudes for  $B^0 \rightarrow \pi^+\pi^-$  and  $\bar{B}^0 \rightarrow \pi^+\pi^-$ , respectively, and similarly for  $A^{00}$ ,  $\bar{A}^{00}$  and  $A^{+0}$ , Grossman and Quinn obtained the bound [7]

$$\cos 2\theta \geq 1 - 2 \frac{B^{00}}{B^{+0}} . \tag{2}$$

In Ref. [8], Charles noted that this bound can be improved:

$$\cos 2\theta \geq \frac{1 - 2B^{00}/B^{+0}}{y} , \tag{3}$$

where  $y \equiv \sqrt{1 - (a_{dir}^{+-})^2}$ . In what follows, we will refer to this as the Grossman-Quinn bound. Charles also pointed out the existence of a second bound:

$$\cos 2\theta \geq \frac{1 - 4B^{00}/B^{+-}}{y} , \tag{4}$$

involving a different ratio of rates. This will be referred to as the Charles bound. From either Eq. (3) or (4), one sees that, given a measurement of  $B^{00}$ , one may be able to put a nontrivial lower bound on  $\cos 2\theta$ , i.e. an upper bound on  $|2\theta|$ . Even if one has only an upper bound on  $B^{00}$ , this still yields a lower limit on  $\cos 2\theta$ . Thus, partial information about  $BR(B^0 \rightarrow \pi^0\pi^0)$  and  $BR(\bar{B}^0 \rightarrow \pi^0\pi^0)$  does indeed allow us to constrain the size of penguin pollution. We note, however, that neither Eq. (3) nor (4) involves all three charge-averaged decay rates,  $B^{+-}$ ,  $B^{+0}$  and  $B^{00}$ . Thus, a condition for the closure of the two isospin triangles is not included in these bounds.

We now turn to our new bound on  $|2\theta|$ , which is the strongest possible bound on this quantity. We assume that the charge-averaged rates  $B^{+-}$  and  $B^{+0}$  have been measured, and that we have (at least) an upper bound on  $B^{00}$ . We will present in detail a geometrical derivation. A second algebraic proof, which gives this bound more directly, will also be outlined.

In the presence of penguin contributions, the  $B \rightarrow \pi\pi$  decay amplitudes take the form

$$\begin{aligned} \frac{1}{\sqrt{2}} A^{+-} &= T e^{i\gamma} + P e^{-i\beta} , \\ A^{00} &= C e^{i\gamma} - P e^{-i\beta} , \\ A^{+0} &= (C + T) e^{i\gamma} , \end{aligned} \tag{5}$$

where the complex amplitudes  $T$ ,  $C$  and  $P$ , which are sometimes referred to as “tree”, “colour-suppressed” and “penguin” amplitudes, include strong phases. Note that we have implicitly imposed the isospin triangle relation

$$\frac{1}{\sqrt{2}}A^{+-} + A^{00} = A^{+0} . \quad (6)$$

The  $\bar{A}$  amplitudes can be obtained from the  $A$  amplitudes by simply changing the signs of the weak phases.

It is convenient to define the new amplitudes  $\tilde{A}^{ij} \equiv e^{2i\gamma}\bar{A}^{ij}$ . Then three observations can be made. First,  $\tilde{A}^{-0} = A^{+0}$ , so that the  $A$  and  $\tilde{A}$  triangles have a common base. (A tiny electroweak penguin amplitude, forming a very small angle between  $A^{+0}$  and  $\tilde{A}^{-0}$ , can be taken into account analytically [9]. However, here it will be neglected.) Second, in the absence of penguin contributions,  $\tilde{A}^{+-} = A^{+-}$ . Thus, the relative phase  $2\theta$  between these two amplitudes is due to penguin pollution. Third, the relative phase between the penguin contributions in  $\tilde{A}^{00}$  and  $A^{00}$  is  $2(\beta + \gamma) \sim 2\alpha$ . All this information is encoded in Fig. 1. Note that the distance between the points  $X$  and  $Y$  is  $2\ell \equiv 2|P|\sin\alpha$ .

Now,  $|P|$  can be expressed in terms of observables [8]:

$$|P|^2 = \frac{B^{+-}}{4\sin^2(\alpha_{eff} - \theta)} [1 - y\cos(2\alpha_{eff} - 2\alpha)] , \quad (7)$$

where  $2\alpha_{eff} = 2\alpha + 2\theta$  is the relative phase between the  $A^{+-}$  and  $e^{-2i\beta}\bar{A}^{+-}$  amplitudes, occurring in the time-dependent rate of  $B^0(t) \rightarrow \pi^+\pi^-$ ,

$$\Gamma(B^0(t) \rightarrow \pi^+\pi^-) = e^{-\Gamma t} B^{+-} \left[ 1 + a_{dir}^{+-} \cos(\Delta mt) - y \sin 2\alpha_{eff} \sin(\Delta mt) \right] . \quad (8)$$

We therefore can write

$$\ell = \frac{1}{2} \sqrt{B^{+-}} \sqrt{1 - y \cos 2\theta} . \quad (9)$$

Thus, a constraint on  $\ell$  implies a bound on  $\cos 2\theta$ .

In order to constrain  $\ell$ , we proceed as follows. First, we assign a coordinate system to Fig. 1 such that the origin is at the midpoint of the points  $X$  and  $Y$ . The points  $X$  and  $Y$  correspond respectively to the coordinates  $(+\ell, 0)$  and  $(-\ell, 0)$ . The points  $W$  and  $Z$  are labelled respectively as  $(x_1, y_1)$  and  $(x_2, y_2)$ . The goal of the exercise is to find the values of these four coordinates. We then note that

$$\begin{aligned} \frac{1}{2} |A^{+-}|^2 &= (x_1 - \ell)^2 + y_1^2 , \\ \frac{1}{2} |\tilde{A}^{+-}|^2 &= (x_1 + \ell)^2 + y_1^2 . \end{aligned} \quad (10)$$

Assuming  $B^{+-}$  and  $a_{dir}^{+-}$  have been measured, we can solve for  $x_1$  and  $y_1$  (up to a discrete ambiguity) as a function of  $\ell$ . We also note that

$$\begin{aligned} |A^{00}|^2 &= (x_2 - \ell)^2 + y_2^2 , \\ |\tilde{A}^{00}|^2 &= (x_2 + \ell)^2 + y_2^2 . \end{aligned} \quad (11)$$

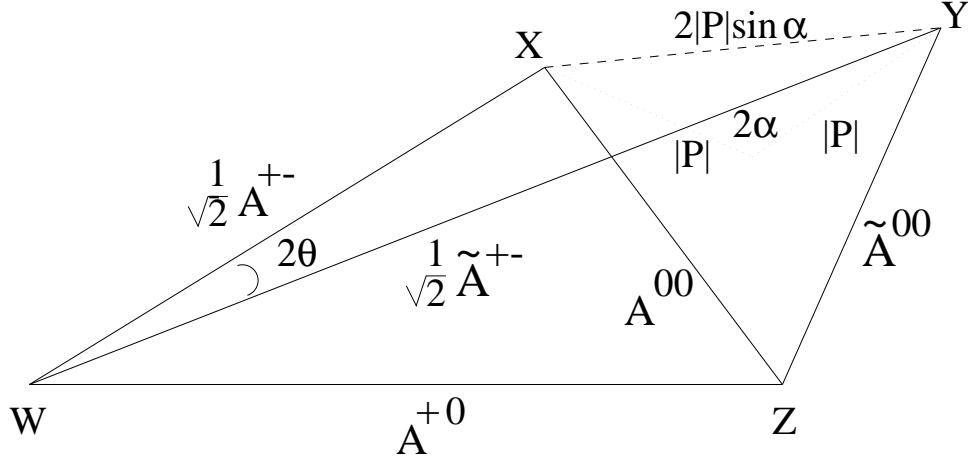


Figure 1: The  $A$  and  $\tilde{A}$  isospin triangles.

Here we assume that only information about  $B^{00}$  is available, so this gives us only one equation in the two unknowns  $x_2$  and  $y_2$ . However, we also have

$$|A^{+0}|^2 = (x_1 - x_2)^2 + (y_1 - y_2)^2, \quad (12)$$

So this gives us another equation involving  $x_2$  and  $y_2$ . We therefore have four (nonlinear) equations in four unknowns, and we can solve for these coordinates as a function of  $\ell$ . The equations are:

$$\begin{aligned} B^{+-} &= 2(x_1^2 + y_1^2) + 2\ell^2, \\ B^{+-}a_{dir}^{+-} &= -4x_1\ell, \\ B^{00} &= (x_2^2 + y_2^2) + \ell^2, \\ B^{+0} &= (x_1^2 + y_1^2) + (x_2^2 + y_2^2) - 2x_1x_2 - 2y_1y_2. \end{aligned} \quad (13)$$

However, the key point is the following: we must obtain only real solutions for  $x_2$  and  $y_2$ , otherwise the triangles do not close. This puts a constraint on  $\ell$ , which, with a bit of simple algebra, can be written

$$x_1^2 C_1^2 - (x_1^2 + y_1^2)(C_1^2 - C_2 y_1^2) \geq 0, \quad (14)$$

where

$$\begin{aligned} C_1 &\equiv \frac{1}{2} \left( \frac{1}{2} B^{+-} - B^{+0} + B^{00} - 2\ell^2 \right), \\ C_2 &\equiv B^{00} - \ell^2. \end{aligned} \quad (15)$$

This leads to

$$\ell^2 \leq \frac{2B^{+-}B^{00} - \left( \frac{1}{2}B^{+-} - B^{+0} + B^{00} \right)^2}{4B^{+0}}, \quad (16)$$

or, using the expression for  $\ell$  in Eq. (9),

$$\cos 2\theta \geq \frac{\left( \frac{1}{2}B^{+-} + B^{+0} - B^{00} \right)^2 - B^{+-}B^{+0}}{B^{+-}B^{+0}y}. \quad (17)$$

This is the new lower bound on  $\cos 2\theta$  (or upper bound on  $|2\theta|$ ). We reiterate that this bound has been derived assuming that the isospin triangles close and have a common base. Thus, to the extent that isospin is violated, whether by electroweak penguin contributions or by  $\pi^0$ - $\eta, \eta'$  mixing [10], the bound will be correspondingly weakened.

Note that this lower bound on  $\cos 2\theta$  can be written

$$\cos 2\theta \geq \frac{1 - 2B^{00}/B^{+0}}{y} + \frac{(B^{+-} - 2B^{+0} + 2B^{00})^2}{4B^{+-}B^{+0}y}. \quad (18)$$

The first term is simply the Grossman-Quinn bound of Eq. (3). Since the second term is always positive, the new bound is stronger than the Grossman-Quinn bound. Similarly, Eq. (17) can be written

$$\cos 2\theta \geq \frac{1 - 4B^{00}/B^{+-}}{y} + \frac{(B^{+-} - 2B^{+0} - 2B^{00})^2}{4B^{+-}B^{+0}y}, \quad (19)$$

where the first term is the Charles bound of Eq. (4). The second term is positive, so that once again the new bound is more constraining than the Charles bound. Note that neither of the previous bounds fully uses the requirement that the isospin triangles close. By contrast, all isospin information has been used in obtaining Eq. (17), so that this is the most stringent possible bound on  $\cos 2\theta$ .

An alternative way of deriving the new bound of Eq. (17) involves the direct calculation of the minimum of  $\cos 2\theta$  under the assumption that  $B^{+-}$ ,  $B^{+0}$ ,  $B^{00}$  and  $y$  are given. Let us define  $\Phi$  to be the angle between  $A^{+-}$  and  $A^{+0}$ , and  $\bar{\Phi}$  to be the angle between  $\tilde{A}^{+-}$  and  $\tilde{A}^{-0}$ . Then  $2\theta$  is equal to  $\Phi + \bar{\Phi}$  or  $\Phi - \bar{\Phi}$ , depending on the relative orientation of the triangles. The minimum of  $\cos 2\theta$  is obviously obtained when the two triangles lie on two opposite sides of  $A^{+0}$ , corresponding to  $2\theta = \Phi + \bar{\Phi}$ . From Fig. 1,  $\cos \Phi$  and  $\cos \bar{\Phi}$  can be expressed in terms of measurable quantities as

$$\begin{aligned} \cos \Phi &= \frac{\frac{1}{2}B^{+-}(1 + a_{dir}^{+-}) + B^{+0} - B^{00}(1 + a_{dir}^{00})}{\sqrt{2}\sqrt{B^{+-}(1 + a_{dir}^{+-})}\sqrt{B^{+0}}}, \\ \cos \bar{\Phi} &= \frac{\frac{1}{2}B^{+-}(1 - a_{dir}^{+-}) + B^{+0} - B^{00}(1 - a_{dir}^{00})}{\sqrt{2}\sqrt{B^{+-}(1 - a_{dir}^{+-})}\sqrt{B^{+0}}}, \end{aligned} \quad (20)$$

where  $a_{dir}^{00}$  is defined analogously to  $a_{dir}^{+-}$  in Eq. (1). By minimizing  $\cos(\Phi + \bar{\Phi})$  with respect to  $a_{dir}^{00}$ , one finds that the minimum is obtained for

$$(a_{dir}^{00})_{min} = \frac{a_{dir}^{+-}}{2} \frac{B^{+-}(\frac{1}{2}B^{+-} - B^{+0} - B^{00})}{B^{00}(\frac{1}{2}B^{+-} + B^{+0} - B^{00})}. \quad (21)$$

The value of  $\cos(\Phi + \bar{\Phi})$  at the minimum is given by the right-hand-side of Eq. (17).

Above we have derived a new upper bound on  $|2\theta|$ . This then raises the question: is it possible to find a lower bound on this quantity? Unfortunately, the answer is no. This can be seen quite clearly in Fig. 1. Suppose that the two-triangle isospin construction can be made for some nonzero value of  $2\theta$ . It is then straightforward to show that one can always rotate  $A^{+-}$  and  $\tilde{A}^{+-}$  continuously around  $W$  towards one

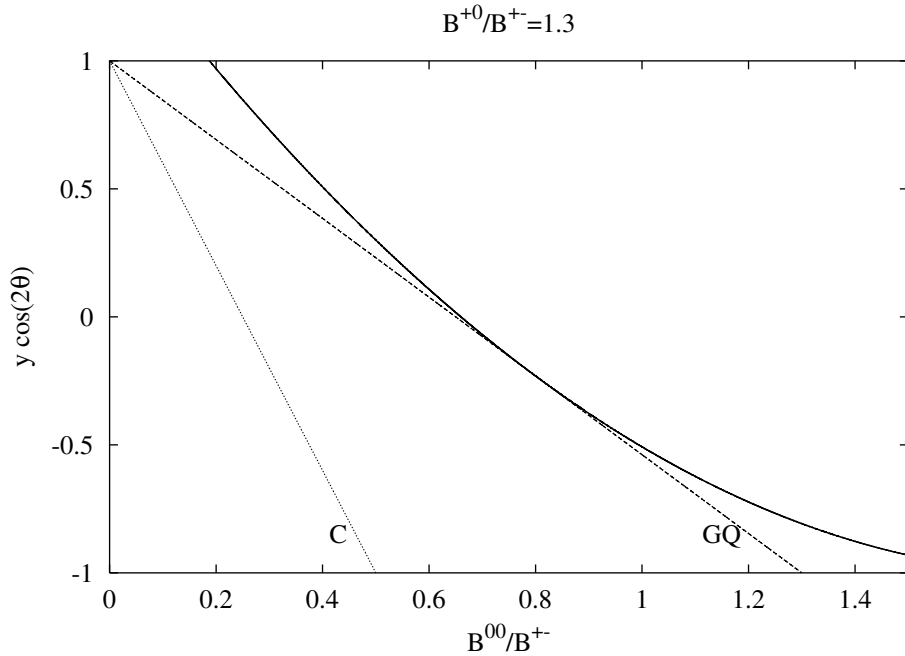


Figure 2: Bounds on  $y \cos 2\theta$  as a function of  $B^{00}/B^{+-}$ , for  $B^{+0}/B^{+-} = 1.3$ . The curves represent the new bound [solid line, Eq. (17)], the Grossman-Quinn bound [dashed line, label GQ, Eq. (3)] and the Charles bound [dotted line, label C, Eq. (4)]. In all cases, the area below the curve is ruled out.

another, without changing  $B^{00}$ , until they lie on one line corresponding to  $\theta = 0$ . Thus, without measuring separately  $B^0 \rightarrow \pi^0\pi^0$  and  $\overline{B}^0 \rightarrow \pi^0\pi^0$ , one cannot put a lower bound on the penguin pollution parameter.

We now turn to a comparison of the new bound on  $\cos 2\theta$  with the Grossman-Quinn and Charles bounds. The present world averages for the  $B \rightarrow \pi\pi$  branching ratios are (in units of  $10^{-6}$ ) [11]:

$$\begin{aligned} BR(B \rightarrow \pi^+\pi^-) &= 4.4 \pm 0.9, \\ BR(B \rightarrow \pi^+\pi^0) &= 5.6 \pm 1.5, \\ BR(B \rightarrow \pi^0\pi^0) &< 5.7 \text{ (90\% C.L.)}. \end{aligned} \quad (22)$$

For the purpose of illustration, we take central values,  $B^{+0}/B^{+-} = 1.3$ , and compare the three bounds in Fig. 2. In this figure, we plot  $y \cos 2\theta$  as a function of  $B^{00}/B^{+-}$ . In all cases, the region of parameter space below the curve is ruled out. As expected, the new bound is (almost) always more stringent than the Grossman-Quinn and Charles bounds. (The new bound is equivalent to the Grossman-Quinn bound when  $2B^{+0}/B^{+-} - 2B^{00}/B^{+-} = 1$  [see Eq. (18)], i.e. for  $B^{00}/B^{+-} = 0.8$ .) Note also that the curves represent the weakest possible lower bound on  $\cos 2\theta$ , obtained for  $y = 1$ . Should  $a_{dir}^{+-}$  be measured to be nonzero (i.e.  $y < 1$ ), this will place a correspondingly stronger lower bound on  $\cos 2\theta$ .

The lower bound on  $\cos 2\theta$  can be straightforwardly converted into an upper bound on  $|\theta|$ . This is shown explicitly in Fig. 3, where we plot the three bounds on

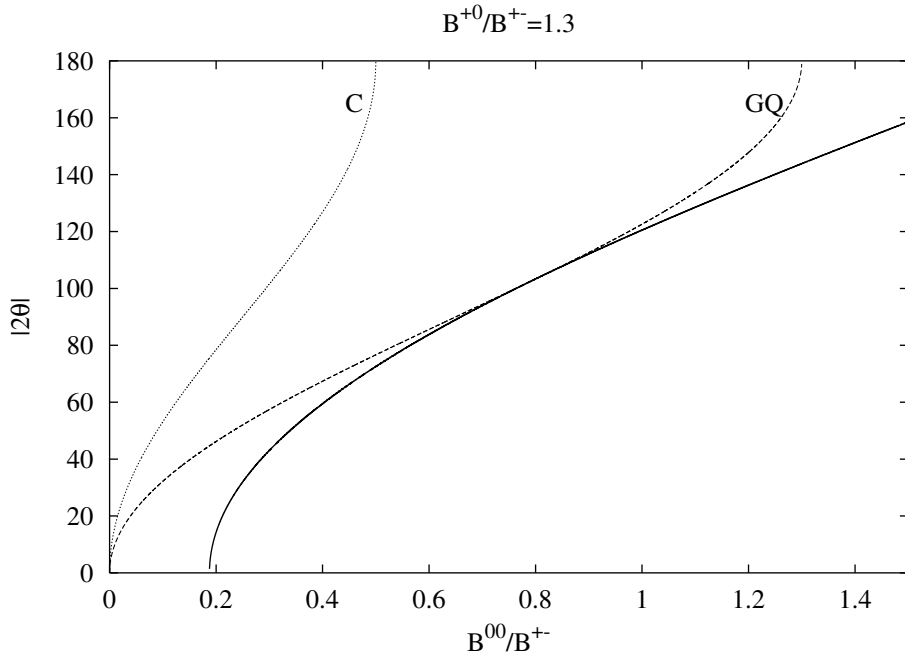


Figure 3: Constraints on  $|2\theta|$  as a function of  $B^{00}/B^{+-}$ , for  $B^{+0}/B^{+-} = 1.3$  and  $y = 1$ . The curves represent the new bound [solid line, Eq. (17)], the Grossman-Quinn bound [dashed line, label GQ, Eq. (3)] and the Charles bound [dotted line, label C, Eq. (4)]. In all cases, values of  $|2\theta|$  above the curve are ruled out.

$|2\theta|$  as a function of  $B^{00}/B^{+-}$ , for  $B^{+0}/B^{+-} = 1.3$  and  $y = 1$ . In all cases, values of  $|2\theta|$  above the curves are excluded.

One interesting feature of Fig. 2 is that the lower bound on  $y \cos 2\theta$  seems to exceed unity for sufficiently small values of  $B^{00}/B^{+-}$ . This implies that there is a lower limit on  $B^{00}/B^{+-}$ , which one cannot see with the Grossman-Quinn or Charles bounds. (This is also seen quite clearly in Fig. 3.) This lower limit, as well as an upper limit on the same quantity, follows directly from the closure of the two isospin triangles, which can be shown to imply that

$$\frac{1}{2} + \frac{B^{+0}}{B^{+-}} - \sqrt{\frac{B^{+0}}{B^{+-}}(1+y)} \leq \frac{B^{00}}{B^{+-}} \leq \frac{1}{2} + \frac{B^{+0}}{B^{+-}} + \sqrt{\frac{B^{+0}}{B^{+-}}(1+y)}. \quad (23)$$

(Equivalently, this constraint can be obtained from Eq. (17) using the condition that  $\cos 2\theta \leq 1$ .) The limits are weakest for  $y = 1$ . For  $B^{+0}/B^{+-} = 1.3$ , one finds  $0.19 \leq B^{00}/B^{+-} \leq 3.4$ . An obvious implication of the closure of the two isospin triangles is that a substantial deviation of  $2B^{+0}/B^{+-}$  from one, as demonstrated by the present central values of  $B^{+-}$  and  $B^{+0}$ , would be evidence for a sizeable value of  $B^{00}$ .

This lower limit on  $B^{00}/B^{+-}$  is useful for two reasons. First, it will give experimentalists some knowledge of the branching ratios for  $B^0/\bar{B}^0 \rightarrow \pi^0\pi^0$ . This in turn will help to anticipate the feasibility of the full isospin analysis. Second, since the bound on  $B^{00}/B^{+-}$  relies only on the closure of the two triangles, it will hold even in the presence of isospin-violating electroweak-penguin contributions. On



the other hand, it has been pointed out by Gardner [10] that the triangles will not close in the presence of other isospin-violating effects such as  $\pi^0$ - $\eta$ ,  $\eta'$  mixing. Thus, the comparison of the actual branching ratio  $B^{00}$  with this bound may give some information about the size of such isospin-violating effects.

Fig. 3 also shows that the upper bound on  $|2\theta|$  deteriorates rather quickly as  $B^{00}/B^{+-}$  increases above its minimum value [Eq. (23)]. This behaviour can be easily understood: writing  $B^{00}/B^{+-} = (B^{00}/B^{+-})_{min} + \Delta B$ , and expanding  $\cos 2\theta$  around  $2\theta = 0$ , Eq. (17) gives, for  $y = 1$ ,

$$(2\theta)^2 \leq \frac{4\sqrt{2}}{\sqrt{B^{+0}/B^{+-}}} \Delta B - \frac{2}{B^{+0}/B^{+-}} \Delta B^2 . \quad (24)$$

For  $B^{+0}/B^{+-}$  of order unity, the coefficient of the linear term is quite large, which causes the rapid deterioration of the upper bound on  $|2\theta|$  as  $\Delta B$  increases. The only way to avoid this is if  $B^{+0}/B^{+-}$  is very large. However, since this possibility is disfavoured experimentally, we conclude that the bound on  $|2\theta|$  will be very weak unless  $B^{00}/B^{+-}$  happens to be near its minimum allowed value. Of course, if  $B^{00}/B^{+-}$  is above its minimum, then it may well be possible to carry out the full isospin analysis.

Although no lower limit can be obtained on the penguin-pollution angle  $|2\theta|$ , we note that a lower bound can be derived for the magnitude of the penguin amplitude  $P$  from measurements of  $B^0(t) \rightarrow \pi^+\pi^-$  alone. Consider again the expression for  $|P|$  given in Eq. (7). This can be minimized with respect to  $\theta$ , yielding

$$\tan \theta|_{minimum} = -\cot \alpha_{eff} \left( \frac{1-y}{1+y} \right) . \quad (25)$$

The minimal value of  $|P|^2$  is then

$$|P|_{min}^2 = \frac{B^{+-}(1-y^2)}{4(1-y \cos 2\alpha_{eff})} , \quad (26)$$

where  $\alpha_{eff}$  is measured in the time-dependent rate of  $B^0(t) \rightarrow \pi^+\pi^-$  [Eq. (8)].

To sum up, in the presence of penguin contributions, the CP asymmetry in  $B^0(t) \rightarrow \pi^+\pi^-$  no longer cleanly measures  $\alpha$ . It is possible to remove this penguin “pollution” with the help of an isospin analysis using all three  $B \rightarrow \pi\pi$  decays and their charge conjugates. However, this analysis requires separate measurements of  $BR(B^0 \rightarrow \pi^0\pi^0)$  and  $BR(\bar{B}^0 \rightarrow \pi^0\pi^0)$ . It will not be possible to carry out the complete isospin analysis if only the sum of this branching ratios can be measured. Nevertheless, we have shown that an upper bound on the penguin-pollution angle  $|2\theta|$  can be obtained. This new bound is an improvement over bounds suggested earlier by Grossman and Quinn, and by Charles. Indeed, since this bound follows from the requirements that the two isospin triangles close and have a common base, it is the most stringent bound possible on  $|2\theta|$ . We have also derived a lower bound on the ratio of branching ratios  $B^{00}/B^{+-}$ . If  $B^{+0}/B^{+-}$  is larger than unity, as is suggested by present central values, then  $B^{00}/B^{+-}$  cannot be tiny and it may be possible to carry out the full isospin analysis. Finally, we have also shown how to obtain a lower bound on the magnitude of the penguin amplitude  $P$ .

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